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NUMERICAL PREDICTION OF RESIDUAL STRESSES IN AN AUTOFRETTAGED TUBE OF COMPRESSIBLE MATERIAL

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The residual stresses in an autofrettaged tube of compressible material are obtained by a new finite-difference approach. The tube is assumed to obey the von Mises' yield criterion, the Prandtl-Reuss flow theory and the isotropic-hardening rule. In order to test the accuracy of the computer program, a convergence study for a nearly incompressible tube has been made and compared with the exact solution as well as the simulated results for residual stresses in an incompressible tube.

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INTRODUCTION

The importance of favorable residual stresses in an autofrettaged tube is well-known.[1] Many methods for predicting residual stresses have been reported.[2-4] For an elastic-plastic material which obeys the von Mises' yield criterion and the associated flow rules, a closed form solution exists only in the plane-strain case neglecting strain hardening and compressibility.[5] Recently a method to simulate this problem by thermal loads was devised by Hussain et al.[6] For a compressible material with or without strain hardening, a new finite-difference approach has been developed by this author.[7] Two types of incremental loadings were discussed in the preceding reference.

In the present paper, the numerical prediction of residual stresses in an autofrettaged tube of compressible material will be reported. The effect of Poisson's ratio will be discussed. In order to test the accuracy of the computer program, a convergence study for a nearly incompressible tube has been made and compared with the exact solution as well as the simulated results for residual stresses in an incompressible tube.

INCOMPRESSIBLE TUBE

For an ideally-plastic incompressible tube which obeys the von Mises' yield criterion and the associated flow rules, a closed form solution exists in the plane-strain case. The residual stresses and displacement after complete elastic unloading in a partially autofrettaged tube are given by,[5]

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*References are listed at the end of this report.

$$\left.\begin{matrix}\sigma_r\\ \sigma_\theta\end{matrix}\right\} = \frac{\sigma_o}{\sqrt{3}}\left[\left(\frac{\rho^2}{b^2} - 2\log\frac{\rho}{r} \mp 1\right) - p_1\left(1 \mp \frac{b^2}{\rho^2}\right)\right] \qquad a < r < \rho \tag{1}$$

$$\left.\begin{matrix}\sigma_r\\ \sigma_\theta\end{matrix}\right\} = \frac{\sigma_o}{\sqrt{3}}\left(\frac{\rho^2}{b^2} - p_1\right)\left(1 \mp \frac{b^2}{\rho^2}\right) \qquad \rho < r < b \tag{2}$$

$$\sigma_z = \begin{cases} \dfrac{\sigma_o}{\sqrt{3}}(\rho^2/b^2 - 2\log\rho/r - p_1) & a < r < \rho \\[2ex] \dfrac{\sigma_o}{\sqrt{3}}(\rho^2/b^2 \qquad\qquad - p_1) & \rho < r < b \end{cases} \tag{3}$$

$$u/r = (\sqrt{3}/2)(\sigma_o/E)(\rho/r)^2 \tag{4}$$

where

$$p_1 = (1 - \rho^2/b^2 + 2\log\rho/a)/(b^2/a^2 - 1) \tag{5}$$

and ρ is the radius of the autofrettaged interface.

According to Hussain et al,[6] the distribution of radial and hoop stresses can be simulated by a steady state thermal loading. The equivalence between the temperature gradient and the yield stress is

$$\frac{E\alpha(T_a - T_\rho)}{2(1-\nu)\log(\rho/a)} = \frac{2\sigma_o}{\sqrt{3}} \tag{6}$$

and the temperature distribution is given by

$$\begin{aligned} T &= T_a - \frac{(T_a - T_\rho)}{\log(\rho/a)}\log(r/a) & a < r < \rho \\ T &= T_\rho & \rho < r < b \end{aligned} \tag{7}$$

FINITE-DIFFERENCE APPROACH

For a compressible material with or without strain hardening, a new finite-difference approach has been developed by this author.[7] An incremental procedure is used for pressure beyond the elastic limit and the elastic solution is used as the initial condition. The cross section of the tube is divided into n rings and we want to determine all incremental quantities at all grid points in each incremental step. In the plastic region, the incremental stresses are related to the incremental strains by the incremental form

$$\Delta\sigma_i = d_{ij}\,\Delta\varepsilon_j \quad \text{for} \quad i,j = r,\theta,z \tag{8}$$

and

$$d_{ij}/2G = \nu/(1-2\nu) + \delta_{ij} - \sigma_i'\sigma_j'/S \tag{9}$$

where E is Young's modulus, ν is Poisson's ratio, δ_{ij} is the Kronecker delta,

$$S = \frac{2}{3}\,(1 + \frac{1}{3}\,H'/G)\sigma^2 \quad , \quad 2G = E/(1+\nu)$$

$$\sigma_m = (\sigma_r+\sigma_\theta+\sigma_z)/3 \quad , \quad \sigma_i' = \sigma_i - \sigma_m$$

$$\sigma = (1/\sqrt{2})[(\sigma_r-\sigma_\theta)^2 + (\sigma_\theta-\sigma_z)^2 + (\sigma_z-\sigma_r)^2]^{1/2} > \sigma_o \tag{10}$$

and σ_o is the yield stress in simple tension or compression. For a strain-hardening material, H' is the slope of the effective stress/plastic strain curve. For an ideally-plastic material, H' = 0. When $\sigma < \sigma_o$ or $d\sigma < 0$, the state of stress is elastic and the third term in equation (9) disappears. Using equation (8) and $\Delta u = r\Delta\varepsilon_\theta$, there exists only two unknowns at each station that have to be determined for each increment of loading. The unknown variables in the present formulation are $(\Delta\varepsilon_\theta)_i$, $(\Delta\varepsilon_r)_i$, for $i = 1,2,\ldots n,n+1$.

The equation of equilibrium and the equation of compatibility are valid for both the elastic and the plastic regions of a thick-walled tube. The finite-difference forms of these two equations at $i = 1,\ldots,n$ are given by

$$(r_{i+1}-2r_i)(\Delta\sigma_r)_i - (r_{i+1}-r_i)(\Delta\sigma_\theta)_i + r_i(\Delta\sigma_r)_{i+1}$$
$$= (r_{i+1}-r_i)(\sigma_\theta-\sigma_r)_i - r_i[(\sigma_r)_{i+1} - (\sigma_r)_i] \qquad (11)$$

for the equation of equilibrium, and

$$(r_{i+1}-2r_i)(\Delta\varepsilon_\theta)_i - (r_{i+1}-r_i)(\Delta\varepsilon_r)_i + r_i(\Delta\varepsilon_\theta)_{i+1}$$
$$= (r_{i+1}-r_i)(\varepsilon_r-\varepsilon_\theta)_i - r_i[(\varepsilon_\theta)_{i+1} - (\varepsilon_\theta)_i] \qquad (12)$$

for the equation of compatibility.

With the aid of the incremental stress-strain relations (equation (8)), equation (11) can be rewritten as

$$[(r_{i+1}-2r_i)(d_{12})_i + (-r_{i+1}+r_i)(d_{22})_i](\Delta\varepsilon_\theta)_i$$
$$+ [(r_{i+1}-2r_i)(d_{11})_i + (-r_{i+1}+r_i)(d_{21})_i](\Delta\varepsilon_r)_i$$
$$+ r_i(d_{12})_{i+1}(\Delta\varepsilon_\theta)_{i+1} + r_i(d_{11})_{i+1}(\Delta\varepsilon_r)_{i+1}$$
$$= (r_{i+1}-r_i)(\sigma_\theta-\sigma_r) - r_i[(\sigma_r)_{i+1} - (\sigma_r)_i] \qquad (13)$$

The boundary conditions for the problem are

$$\Delta\sigma_r(a,t) = -\Delta p \quad , \quad \Delta\sigma_r(b,t) = 0 \qquad (14)$$

Using the incremental relations (equation (8)), we rewrite equation (11) as

$$(d_{12})_1(\Delta\varepsilon_\theta)_1 + (d_{11})_1(\Delta\varepsilon_r)_1 = -\Delta p \qquad (15)$$

and

$$(d_{12})_{n+1}(\Delta\varepsilon_\theta)_{n+1} + (d_{11})_{n+1}(\Delta\varepsilon_r)_{n+1} = 0 \qquad (16)$$

Now we can form a system of $2(n+1)$ equations for solving $2(n+1)$ unknowns, $(\Delta\varepsilon_\theta)_i$, $(\Delta\varepsilon_r)_i$, for $i = 1,2,\ldots,n,n+1$. Equations (15) and (16) are taken as the first and last equations, respectively, and the other $2n$ equations are set up at $i = 1,2,\ldots,n$ using equations (12) and (13). The final system is an unsymmetric band matrix with the nonzero terms clustered about the main diagonal, two below and one above.

When the total applied pressure p is given, it is natural to divide the loading path into m equal fixed increments with $\Delta p = (p-p^*)/m$ where p^* is the pressure corresponding to initial yielding. These fixed increments need not be equal for all steps and any sequence of m increments can be supplied by the user. In Reference 7, an adaptive algorithm to generate a sequence of load increments was described.

NUMERICAL RESULTS AND DISCUSSIONS

In order to test the accuracy of the computer program, a convergence study for a nearly incompressible tube ($\nu = .4999999$) has been made and compared with the exact solution for an incompressible tube ($\nu = 1/2$). The numerical results for a tube with $b/a = 2$ and $H' = 0$ are very accurate as shown in Table I for 30, 60, and 100 percent overstrain. A comparison of the calculated residual hoop stresses with the exact solution as well as the simulated results is shown in Table II. The finite-difference approach can generate more accurate results than the method of simulation by thermal load for incompressible material.

In order to discuss the effect of compressibility, we calculated the residual stresses for a tube with $b/a = 2$, $H' = 0$, $n = 400$, $\nu = 0, 0.3, 0.4999$. The results are shown in Tables III, IV, and V for residual hoop, radial, and axial components, respectively. The effect of hardening on the residual stresses can be discussed in a similar way. The results for a tube with $b/a = 2$, $\nu = 0.3$, $n = 400$, $H'/E = 0, 1/9, 1/19$ ($w = Et/E = 0, 0.05, 0.0$) are shown in Tables VI, VII, and VIII for residual hoop, radial, and axial components, respectively. It can be seen that the effect of hardening on residual hoop stress is larger than that of compressibility.

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TABLE I. CONVERGENCE STUDY FOR A NEARLY INCOMPRESSIBLE TUBE UNDER INTERNAL PRESSURE ($b/a = 2$, $H' = 0$, $\nu = .4999999$)

O.S.	n	P/σ_o	MAX σ_θ/σ_o	Inside σ_z/σ_o	$\frac{E}{\sigma_o}\frac{U_a}{a}$
30%	10	.64630	.80697	-.06895	1.54781
	20	.64099	.81444	-.06364	1.50104
	50	.63815	.81861	-.05080	1.47764
	100	.63725	.81996	-.05990	1.47047
	200	.63681	.82062	-.05946	1.46699
	400	.63659	.82095	-.05924	1.46528
	*	.63637	.82128	-.05902	1.46358
60%	10	.77375	.93345	-.19640	2.49329
	20	.76123	.94049	-.18388	2.33897
	50	.75464	.94438	-.17729	2.26259
	100	.75257	.94563	-.17522	2.23922
	200	.75156	.94625	-.17421	2.22805
	400	.75105	.94655	-.17371	2.22251
	*	.75056	.94685	-.17321	2.21703
100%	10	.82096	1.15470	-.24361	4.14111
	20	.80999	1.15470	-.23264	3.76669
	50	.80408	1.15470	-.22673	3.57791
	100	.80221	1.15470	-.22486	3.51990
	200	.80129	1.15470	-.22394	3.49173
	400	.80083	1.15470	-.22348	3.47785
	*	.80038	1.15470	-.22303	3.46410

* Exact solution.

TABLE II. A COMPARISON OF RESIDUAL HOOP STRESS (σ_θ/σ_o) FOR b/a = 2, H' = 0

O.S.	r/a	ν = .5 Exact	n = 400 ν = .4999	ν = .3000 Simulation
30%	1.0	-0.54224	-0.54317	-0.54645
	1.1	- .28497	- .28582	- .29157
	1.2	- .07250	- .07329	- .08021
	1.3	+ .10709	+ .10636	+ .09897
	1.4	+ .09672	+ .09587	+ .08962
	1.5	+ .08835	+ .08774	+ .08205
	1.6	+ .8150	+ .08056	+ .07582
	1.7	+ .07583	+ .07487	+ .07065
	1.8	+ .07107	+ .07102	+ .06630
	1.9	+ .06705	+ .06610	+ .06261
	2.0	+ .06361	+ .06267	+ .05945
60%	1.0	-0.84679	-0.84865	-0.85480
	1.1	- .56305	- .56468	- .57384
	1.2	- .33048	- .33191	- .34250
	1.3	- .13525	- .13652	- .14766
	1.4	+ .03190	+ .03076	+ .01955
	1.5	+ .17737	+ .17635	+ .16534
	1.6	+ .30575	+ .30483	+ .29416
	1.7	+ .28446	+ .28345	+ .21408
	1.8	+ .26662	+ .26555	+ .25270
	1.9	+ .25152	+ .25042	+ .24288
	2.0	+ .23863	+ .23752	+ .23062
100%	1.0	-0.97964	-0.98130	-0.99326
	1.1	- .68437	- .68579	- .70058
	1.2	- .44303	- .44425	- .46027
	1.3	- .24098	- .24203	- .25841
	1.4	- .06842	- .06933	- .08559
	1.5	+ .08142	+ .08063	+ .06474
	1.6	+ .21338	+ .21268	+ .19729
	1.7	+ .33099	+ .33037	+ .31553
	1.8	+ .43681	+ .43634	+ .42205
	1.9	+ .53306	+ .53259	+ .51886
	2.0	+ .62111	+ .62069	+ .60749

TABLE III. THE EFFECT OF COMPRESSIBILITY ON THE RESIDUAL STRESS σ_θ/σ_0

($b/a = 2$, $H' = 0$, $n = 400$)

O.S.	r/a	ν = .4999	ν = .3000	ν = .0000
30%	1.0	-0.54317	-0.53992	-0.51455
	1.1	- .28528	- .28233	- .25808
	1.2	- .07329	- .07127	- .05712
	1.3	+ .10636	+ .16389	+ .09593
	1.4	+ .09587	+ .09358	+ .08647
	1.5	+ .08774	+ .08530	+ .07887
	1.6	+ .08056	+ .07854	+ .07266
	1.7	+ .07487	+ .07297	+ .06753
	1.8	+ .07012	+ .06831	+ .06324
	1.9	+ .06610	+ .06437	+ .05962
	2.0	+ .06267	+ .06102	+ .05653
60%	1.0	-0.84865	-0.84138	-0.80090
	1.1	- .56468	- .55776	- .51977
	1.2	- .33191	- .32513	- .28850
	1.3	- .13652	- .13036	- .09892
	1.4	+ .03076	+ .03487	+ .05298
	1.5	+ .17635	+ .17160	+ .17167
	1.6	+ .30483	+ .29721	+ .26278
	1.7	+ .28345	+ .27635	+ .24434
	1.8	+ .26555	+ .25889	+ .22892
	1.9	+ .25042	+ .24414	+ .21587
	2.0	+ .23752	+ .23155	+ .20474
100%	1.0	-0.98130	-0.97388	-0.92931
	1.1	- .68579	- .67902	- .63864
	1.2	- .44425	- .43792	- .40015
	1.3	- .24203	- .23600	- .20018
	1.4	- .06933	- .06370	- .03171
	1.5	+ .08063	+ .08531	+ .10837
	1.6	+ .21268	+ .21530	+ .22222
	1.7	+ .33037	+ .32918	+ .31266
	1.8	+ .43634	+ .42900	+ .38327
	1.9	+ .53259	+ .51654	+ .43768
	2.0	+ .62069	+ .59296	+ .47918

TABLE IV. THE EFFECT OF COMPRESSIBILITY ON THE RESIDUAL STRESS σ_r/σ_o

($b/a = 2$, $H' = 0$, $n = 400$)

O.S.	r/a	ν = .4999	ν = .3000	ν = .0000
30%	1.0	0.00000	0.00000	0.00000
	1.1	- .03732	- .03684	- .25808
	1.2	- .04891	- .04812	- .04462
	1.3	- .04368	- .04287	- .03940
	1.4	- .03320	- .03255	- .02994
	1.5	- .02477	- .02427	- .02234
	1.6	- .01789	- .01752	- .01613
	1.7	- .01220	- .01194	- .01100
	1.8	- .00744	- .00728	- .00671
	1.9	- .00342	- .00335	- .00309
	2.0	0.00000	0.00000	0.00000
60%	1.0	0.00000	0.00000	0.00000
	1.1	- .06371	- .66302	- .05957
	1.2	- .09539	- .09415	- .08795
	1.3	- .10581	- .10413	- .09578
	1.4	- .10182	- .09986	- .09032
	1.5	- .08797	- .08598	- .07658
	1.6	- .06731	- .06566	- .05803
	1.7	- .04593	- .04480	- .03960
	1.8	- .02804	- .02735	- .02417
	1.9	- .01291	- .01259	- .01113
	2.0	0.00000	0.00000	0.00000
100%	1.0	0.00000	0.00000	0.00000
	1.1	- .07520	- .07453	- .07076
	1.2	- .11561	- .11444	- .10780
	1.3	- .13282	- .13125	- .12233
	1.4	- .13424	- .13235	- .12165
	1.5	- .12474	- .12262	- .11079
	1.6	- .10764	- .10541	- .09335
	1.7	- .08523	- .08307	- .07197
	1.8	- .05911	- .05728	- .04850
	1.9	- .03042	- .02929	- .02423
	2.0	0.00000	0.00000	0.00000

TABLE V. THE EFFECT OF COMPRESSIBILITY ON THE RESIDUAL STRESS σ_z/σ_o

(b/a = 2, H' = 0, n = 400)

O.S.	r/a	ν = .4999	ν = .3000	ν = .0000
30%	1.0	-0.27153	-0.15819	+0.01264
	1.1	- .16153	- .08015	+ .03964
	1.2	- .06108	- .02272	+ .02990
	1.3	+ .03133	+ .01831	+ .00000
	1.4	+ .03133	+ .01831	+ .00000
	1.5	+ .03133	+ .01831	+ .00000
	1.6	+ .03133	+ .01831	+ .00000
	1.7	+ .03133	+ .01831	+ .00000
	1.8	+ .03133	+ .01831	+ .00000
	1.9	+ .03133	+ .01831	+ .00000
	2.0	+ .03133	+ .01831	+ .00000
60%	1.0	-0.42426	-0.28532	-0.07295
	1.1	- .31413	- .18422	+ .01377
	1.2	- .21360	- .10266	+ .06134
	1.3	- .12112	- .03886	+ .07385
	1.4	- .03551	+ .00946	+ .06108
	1.5	+ .04419	+ .04477	+ .03373
	1.6	+ .11874	+ .06946	+0.00000
	1.7	+ .11874	+ .06946	+0.00000
	1.8	+ .11874	+ .06946	+0.00000
	1.9	+ .11874	+ .06946	+0.00000
	2.0	+ .11874	+ .06946	+0.00000
100%	1.0	-0.49052	-0.36683	-0.16290
	1.1	- .38037	- .25538	- .05105
	1.2	- .27984	- .15795	+ .03647
	1.3	- .18737	- .07470	+ .09527
	1.4	- .10177	- .00538	+ .12490
	1.5	- .02210	+ .05073	+ .12951
	1.6	+ .05243	+ .09474	+ .11609
	1.7	+ .12243	+ .12800	+ .09177
	1.8	+ .18842	+ .15179	+ .06210
	1.9	+ .25084	+ .16184	+ .03078
	2.0	+ .31028	+ .17789	+ .00000

TABLE VI. THE EFFECT OF HARDENING ON THE RESIDUAL STRESS σ_θ/σ_o

$(b/a = 2,\ \nu = .3,\ n = 400)$

O.S.	r/a	w = 0.00	w = 0.05	w = 0.10
30%	1.0	-0.53992	-0.50612	-0.47446
	1.1	- .28233	- .26457	- .24824
	1.2	- .07127	- .06644	- .06254
	1.3	+ .16389	+ .09831	+ .09216
	1.4	+ .09358	+ .08861	+ .08306
	1.5	+ .08530	+ .08082	+ .07575
	1.6	+ .07854	+ .07445	+ .06978
	1.7	+ .07297	+ .06919	+ .06485
	1.8	+ .06831	+ .06480	+ .06073
	1.9	+ .06437	+ .06108	+ .05725
	2.0	+ .06102	+ .05792	+ .05428
60%	1.0	-0.84138	-0.78984	-0.74017
	1.1	- .55776	- .52382	- .49114
	1.2	- .32513	- .30556	- .28679
	1.3	- .13036	- .12276	- .11559
	1.4	+ .03487	+ .03245	+ .02989
	1.5	+ .17610	+ .16532	+ .15461
	1.6	+ .29721	+ .27952	+ .26204
	1.7	+ .27635	+ .25991	+ .24364
	1.8	+ .25889	+ .24349	+ .22825
	1.9	+ .24414	+ .22962	+ .21524
	2.0	+ .23155	+ .21778	+ .20414
100%	1.0	-0.97388	-0.91430	-0.85591
	1.1	- .67902	- .63781	- .59733
	1.2	- .43792	- .41165	- .38574
	1.3	- .23600	- .22219	- .20843
	1.4	- .06370	- .06050	- .05708
	1.5	+ .08531	+ .07938	+ .07388
	1.6	+ .21530	+ .20147	+ .18825
	1.7	+ .32918	+ .30854	+ .28866
	1.8	+ .42906	+ .40260	+ .37700
	1.9	+ .51634	+ .48518	+ .45473
	2.0	+ .59296	+ .55752	+ .52301

TABLE VII. THE EFFECT OF HARDENING ON THE RESIDUAL STRESS σ_r/σ_o

($b/a = 2$, $\nu = .3$, $n = 400$)

O.S.	r/a	w = 0.00	w = 0.05	w = 0.10
30%	1.0	0.00000	0.00000	-0.00000
	1.1	-0.03684	- .03467	- .03250
	1.2	-0.04812	- .04531	- .04249
	1.3	-0.04287	- .04039	- .03788
	1.4	-0.03255	- .03070	- .02879
	1.5	- .02427	- .02290	- .02149
	1.6	- .01752	- .01654	- .01551
	1.7	- .01194	- .01128	- .01057
	1.8	- .00728	- .00688	- .00645
	1.9	- .00335	- .00317	- .00297
	2.0	0.00000	0.00000	-0.00000
60%	1.0	0.00000	0.00000	-0.00000
	1.1	-0.06302	- .05919	- .05544
	1.2	- .09415	- .08844	- .08286
	1.3	- .10413	- .09784	- .09169
	1.4	- .09986	- .09386	- .08799
	1.5	- .08598	- .08084	- .07580
	1.6	- .06566	- .06174	- .05790
	1.7	- .04480	- .04213	- .03951
	1.8	- .02735	- .02572	- .02411
	1.9	- .01259	- .01184	- .01110
	2.0	- .00000	0.00000	-0.00000
100%	1.0	.00000	0.00000	-0.00000
	1.1	-0.07453	-0.06991	- .06543
	1.2	- .11444	-0.10736	- .10050
	1.3	- .13125	-0.12316	- .11530
	1.4	- .13235	-0.12421	- .11631
	1.5	- .12262	-0.11511	- .10781
	1.6	- .10541	0.09898	- .09272
	1.7	- .08307	- .07803	- .07312
	1.8	- .05728	- .05382	- .05045
	1.9	.02929	- .02753	- .02582
	2.0	0.00000	0.00000	0.00000

TABLE VIII. THE EFFECT OF HARDENING ON THE RESIDUAL STRESS σ_z/σ_o

(b/a = 2, ν = .3, n = 400)

O.S.	r/a	w = 0.00	w = 0.05	w = 0.10
30%	1.0	-0.15819	-0.14544	-0.13389
	1.1	- .08015	- .07403	- .06852
	1.2	- .02272	- .02097	- .01953
	1.3	+ .01831	+ .01737	+ .01628
	1.4	+ .01831	+ .01737	+ .01628
	1.5	+ .01831	+ .01737	+ .01628
	1.6	+ .01831	+ .01737	+ .01628
	1.7	+ .01831	+ .01737	+ .01628
	1.8	+ .01831	+ .01737	+ .01628
	1.9	+ .01831	+ .01737	+ .01628
	2.0	+ .01831	+ .01737	+ .01628
60%	1.0	-0.28532	- .25926	-0.23525
	1.1	- .18422	- .16735	- .15185
	1.2	- .10266	- .09318	- .08450
	1.3	- .03886	- .03494	- .03142
	1.4	+ .00946	+ .00946	+ .00931
	1.5	+ .00447	+ .04219	+ .03959
	1.6	+ .06946	+ .06533	+ .06124
	1.7	+ .06946	+ .06533	+ .06124
	1.8	+ .06946	+ .06533	+ .06124
	1.9	+ .06946	+ .06533	+ .06124
	2.0	+ .06946	+ .06533	+ .06124
100%	1.0	-0.36683	-0.33011	-0.29555
	1.1	- .25538	- .22763	- .20188
	1.2	- .15795	-0.13872	- .12109
	1.3	- .07470	-0.06310	- .05265
	1.4	- .00538	-0.00025	+0.00416
	1.5	+ .05073	+0.05065	- .05022
	1.6	+ .09474	+0.09068	- .08655
	1.7	+ .12800	+0.12107	- .11426
	1.8	+ .15197	+0.14312	- .13449
	1.9	+ .16814	+0.15811	- .14835
	2.0	+ .17789	+0.16726	+ .15690

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